

CLAIMS**What Is Claimed:**

- 1 1. A method for simulating a multi-dimensional space, comprising:
  - 2 generating a sequence of pseudo-random numbers according to a
  - 3 prescribed quasi-Monte Carlo model; and
  - 4 mapping each pseudo-random number R of the sequence of
  - 5 random numbers into multiple variables of unique values for the multi-
  - 6 dimensional space, the multi-dimensional space including D dimensions,
  - 7 where D is a number.
- 1 2. The method of claim 1, further comprising assigning the unique values to
- 2 each dimension based upon a prescribed index.
- 1 3. The method of claim 1, further comprising sampling the multiple variables
- 2 of the multi-dimensional space and statistically analyzing the sampled
- 3 multiple variables according to a prescribed error analysis.
- 1 4. The method of claim 1, further comprising sampling the multiple variables
- 2 of the multi-dimensional space and performing numerical integrations
- 3 upon the sampled multiple variables.
- 1 5. The method of claim 1, wherein each pseudo-random number R
- 2 generated by the prescribed quasi-Monte Carlo model includes a floating
- 3 point number having a value between 0.0 and 1.0, further wherein each
- 4 dimension is characterized by a unique value based upon an index, the
- 5 index equal to a total combinations of dimensional value points TC times a
- 6 respective pseudo-random number R.

1    6. The method of claim 1, wherein each of the multiple variables of the multi-  
2    dimensional space represents a corresponding D dimension value and  
3    wherein each dimension is characterized by a minimum and a maximum  
4    value, further wherein each dimension is characterized by a prescribed  
5    resolution S.

1    7. The method of claim 6, wherein the D dimension values are further  
2    characterized by a first dimension D0 that includes minimum and  
3    maximum values defined as D0.min and D0.max, respectively, a second  
4    dimension D1 that includes minimum and maximum values defined as  
5    D1.min and D1.max, etceteras, up to a Dth dimension.

1    8. The method of claim 6, further comprising selecting a value of S according  
2    to a desired accuracy of a final simulation value, wherein the value of S  
3    defines a grid for use in conjunction with the mapping of the pseudo-  
4    random numbers into the multiple variables of the multi-dimensional  
5    space.

1    9. The method of claim 8, wherein selecting the value of S includes deriving  
2    the value of S such that a ratio r, as defined by  $r = s^D/P^N$ , is not factorable  
3    by one of the following selected from the group consisting of base P and  
4    the number of dimensions D, and where N is the number of pseudo-  
5    random numbers and r is a prescribed prime number.

1 10. A method for simulating a multi-dimensional space, comprising:

2 generating a sequence of pseudo-random numbers according to a  
3 prescribed quasi-Monte Carlo model;

4 mapping each pseudo-random number R of the sequence of  
5 random numbers into multiple variables of unique values for the multi-  
6 dimensional space, the multi-dimensional space including D dimensions,  
7 wherein D is a number, wherein each of the multiple variables of the multi-  
8 dimensional space represents a corresponding D dimension value and  
9 wherein each dimension is characterized by a minimum and a maximum  
10 value, the D dimension values further being characterized by a first  
11 dimension D0 that includes minimum and maximum values defined as  
12 D0.min and D0.max, respectively, a second dimension D1 that includes  
13 minimum and maximum values defined as D1.min and D1.max, etceteras,  
14 up to a Dth dimension, further wherein each dimension is characterized by  
15 a prescribed resolution S; and

16 selecting a value of S according to a desired accuracy of a final  
17 simulation value, wherein the value of S defines a grid for use in  
18 conjunction with the mapping of the pseudo-random numbers into the  
19 multiple variables of the multi-dimensional space, wherein selecting the  
20 value of S includes deriving the value of S such that a ratio r, as defined  
21 by  $r = s^D/P^N$ , is not factorable by one of the following selected from the  
22 group consisting of base P and the number of dimensions D, and where N  
23 is the number of pseudo-random numbers and r is a prescribed prime  
24 number.

1 11. A method for simulating trace impedance of a printed circuit board  
2 characterized by at least three dimensions of a multi-dimensional space,  
3 said method comprising:  
4 generating a sequence of pseudo-random numbers according to a  
5 prescribed quasi-Monte Carlo model; and  
6 mapping each pseudo-random number R of the sequence of  
7 random numbers into multiple variables of unique values for the multi-  
8 dimensional space, the multi-dimensional space including D dimensions,  
9 where D is a number.

1 12. The method of claim 11, further comprising assigning the unique values to  
2 each dimension based upon a prescribed index.

1 13. The method of claim 11, further comprising sampling the multiple variables  
2 of the multi-dimensional space and statistically analyzing the sampled  
3 multiple variables according to a prescribed error analysis.

1 14. The method of claim 11, further comprising sampling the multiple variables  
2 of the multi-dimensional space and performing numerical integrations  
3 upon the sampled multiple variables.

1 15. The method of claim 11, wherein each pseudo-random number R  
2 generated by the prescribed quasi-Monte Carlo model includes a floating  
3 point number having a value between 0.0 and 1.0, further wherein each  
4 dimension is characterized by a unique value based upon an index, the  
5 index equal to a total combinations of dimensional value points TC times a  
6 respective pseudo-random number R.

1 16. The method of claim 11, wherein each of the multiple variables of the  
2 multi-dimensional space represents a corresponding D dimension value  
3 and wherein each dimension is characterized by a minimum and a  
4 maximum value, further wherein each dimension is characterized by a  
5 prescribed resolution S.

1 17. The method of claim 16, wherein the D dimension values are further  
2 characterized by a first dimension D0 that includes minimum and  
3 maximum values defined as D0.min and D0.max, respectively, a second  
4 dimension D1 that includes minimum and maximum values defined as  
5 D1.min and D1.max, etceteras, up to a Dth dimension.

1 18. The method of claim 16, further comprising selecting a value of S  
2 according to a desired accuracy of a final simulation value, wherein the  
3 value of S defines a grid for use in conjunction with the mapping of the  
4 pseudo-random numbers into the multiple variables of the multi-  
5 dimensional space.

1 19. The method of claim 18, wherein selecting the value of S includes deriving  
2 the value of S such that a ratio r, as defined by  $r = s^D/P^N$ , is not factorable  
3 by one of the following selected from the group consisting of base P and  
4 the number of dimensions D, and where N is the number of pseudo-  
5 random numbers and r is a prescribed prime number.

1       20. Apparatus for simulating trace impedance of a printed circuit board, the  
2       printed circuit board characterized by at least three dimensions of a multi-  
3       dimensional space, said apparatus comprising:

4                 a random number generator for generating a sequence of pseudo-  
5       random numbers according to a prescribed quasi-Monte Carlo model;

6                 a mapping processor for mapping each pseudo-random number R  
7       of the sequence of random numbers into multiple variables of unique  
8       values for the multi-dimensional space, the multi-dimensional space  
9       including D dimensions, where D is a number, wherein each of the  
10      multiple variables of the multi-dimensional space represents a  
11      corresponding D dimension value and wherein each dimension is  
12      characterized by a minimum and a maximum value, the D dimension  
13      values further being characterized by a first dimension D0 that includes  
14      minimum and maximum values defined as D0.min and D0.max,  
15      respectively, a second dimension D1 that includes minimum and  
16      maximum values defined as D1.min and D1.max, etceteras, up to a Dth  
17      dimension, further wherein each dimension is characterized by a  
18      prescribed resolution S; and

19                 a value selector for selecting a value of S according to a desired  
20      accuracy of a final simulation value, wherein the value of S defines a grid  
21      for use in conjunction with the mapping of the pseudo-random numbers  
22      into the multiple variables of the multi-dimensional space, wherein  
23      selecting the value of S includes deriving the value of S such that a ratio r,  
24      as defined by  $r = s^D/P^N$ , is not factorable by one of the following selected  
25      from the group consisting of base P and the number of dimensions D, and  
26      where N is the number of pseudo-random numbers and r is a prescribed  
27      prime number.

1    21. A method of manufacturing a printed circuit board comprising:

2                 characterizing the printed circuit board by at least three dimensions  
3                 of a multi-dimensional space; and

4                 manufacturing the printed circuit board in accordance with a  
5                 simulated trace impedance, the simulated trace impedance obtained by:

6                         generating a sequence of pseudo-random numbers  
7                         according to a prescribed quasi-Monte Carlo model;

8                         mapping each pseudo-random number R of the sequence of  
9                 random numbers into multiple variables of unique values for the  
10                 multi-dimensional space, the multi-dimensional space including D  
11                 dimensions, where D is a number, wherein each of the multiple  
12                 variables of the multi-dimensional space represents a  
13                 corresponding D dimension value and wherein each dimension is  
14                 characterized by a minimum and a maximum value, the D  
15                 dimension values further being characterized by a first dimension  
16                 D0 that includes minimum and maximum values defined as D0.min  
17                 and D0.max, respectively, a second dimension D1 that includes  
18                 minimum and maximum values defined as D1.min and D1.max,  
19                 etceteras, up to a Dth dimension, further wherein each dimension is  
20                 characterized by a prescribed resolution S; and

21                 selecting a value of S according to a desired accuracy of a  
22                 final simulation value, wherein the value of S defines a grid for use  
23                 in conjunction with the mapping of the pseudo-random numbers  
24                 into the multiple variables of the multi-dimensional space, wherein  
25                 selecting the value of S includes deriving the value of S such that a  
26                 ratio r, as defined by  $r = s^D/P^N$ , is not factorable by one of the  
27                 following selected from the group consisting of base P and the  
28                 number of dimensions D, and where N is the number of pseudo-  
29                 random numbers and r is a prescribed prime number.

1 22. A computer system, comprising:

2           a printed circuit board manufactured in accordance with a simulated  
3           trace impedance, said printed circuit board including impedance traces  
4           that characterize at least three dimensions of a multi-dimensional space of  
5           said printed circuit board , wherein said impedance traces include trace  
6           impedances obtained by:

7               generating a sequence of pseudo-random numbers  
8               according to a prescribed quasi-Monte Carlo model;

9               mapping each pseudo-random number R of the sequence of  
10          random numbers into multiple variables of unique values for the  
11          multi-dimensional space, the multi-dimensional space including D  
12          dimensions, where D is a number, wherein each of the multiple  
13          variables of the multi-dimensional space represents a  
14          corresponding D dimension value and wherein each dimension is  
15          characterized by a minimum and a maximum value, the D  
16          dimension values further being characterized by a first dimension  
17          D0 that includes minimum and maximum values defined as D0.min  
18          and D0.max, respectively, a second dimension D1 that includes  
19          minimum and maximum values defined as D1.min and D1.max,  
20          etceteras, up to a Dth dimension, further wherein each dimension is  
21          characterized by a prescribed resolution S; and

22               selecting a value of S according to a desired accuracy of a  
23          final simulation value, wherein the value of S defines a grid for use  
24          in conjunction with the mapping of the pseudo-random numbers  
25          into the multiple variables of the multi-dimensional space, wherein  
26          selecting the value of S includes deriving the value of S such that a  
27          ratio r, as defined by  $r = s^D/P^N$ , is not factorable by one of the  
28          following selected from the group consisting of base P and the  
29          number of dimensions D, and where N is the number of pseudo-  
30          random numbers and r is a prescribed prime number.